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ELECTROMAGNETIC FIELDS GENERATED BY OCEAN WAVES

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Physical Dynamics, Incorporated

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ELECTROMAGNETIC FIELDS
GENERATED BY OCEAN WAVES

Walter Podney

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ABSTRACT

The electromagnetic field generated by a progressive ocean wave in a horizontally stratified ocean is a sum of a transverse electric type field, a transverse magnetic type field, and an electrostatic type field. Sea water velocity components in a vertical plane containing the direction of wave propagation generate the transverse electric part of the field, and the velocity component normal to the plane generates the transverse magnetic part of the field, which vanishes above an ocean surface. The electrostatic part of the field results from surface charges that halt vertical electric currents at an ocean surface.

Gradients of magnetic fields induced at the surface by both surface and internal waves provide sensible signals for recently developed magnetic gradiometers based on the Josephson effect provided surface speeds exceed 1 cm/sec or so. Vertically spaced measurements of noise spectra of magnetic field gradients above an ocean surface offer a unique and promising means of obtaining a measure of surface and internal wave spectra for three reasons: (1) magnetic field strengths above the surface are proportional to a weighted average of sea water speed over an ocean depth, which provides a response depending on the mode structure of internal waves, (2) field strengths decrease exponentially with height above the surface as e^{-kh}, where k denotes wave number, which provides a means of wave number discrimination and, together with frequency discrimination, offers means of resolving internal wave spectra mode by mode, and (3) noise spectra of magnetic field gradients are effectively cross-spectra and so provide directional information on wave spectra from an effective point measurement.

INTRODUCTION

As Faraday [1832] first recognized, sea water moving across the Earth's steady magnetic field induces eddy currents in a sea that generate an electromagnetic field characteristic of the velocity field in the sea. First investigations of the effect were concerned with measurement of electric fields induced by steady ocean currents. Efforts culminated in development of a so-called geomagnetic electrokinetographic or GEK method [von Arx, 1950] of inferring velocities of broad, steady, ocean currents from measurements of horizontal differences in electric potential at an ocean surface.

Longuet-Higgins, et al. [1954] develop a sound basis for interpreting GEK measurements and consider electric fields and currents produced by surface waves as well as steady currents. Investigation of magnetic fields generated by either steady ocean currents or ocean waves, however, has been largely neglected until recently. Larsen [1968] and Sanford [1971] among others investigate electric and magnetic fields generated by deep sea tides and other large scale, quasistatic, ocean currents. Crews and Futterman [1962] are the first to investigate magnetic fields produced by surface waves. They neglect magnetic induction effects and, by using the integral form of Ampere's law, determine magnetic fields produced above the surface by waves on the surface of a deep ocean. Warburton and Caminiti [1964] extend the analysis to determine magnetic fields produced below the surface of a deep ocean, and Groskaya, et al. [1972] determine magnetic fields produced by surface waves in shallow seas. Weaver [1965] is the first to account for magnetic

induction effects in determining magnetic fields induced by surface waves on a deep ocean by using differential forms of both Faraday's and Ampere's laws. Beal and Weaver [1970] also investigate magnetic fields produced by internal waves propagating along a sharp thermocline. Nonetheless, their investigations are restricted to irrotational velocity fields, which is a suitable approximation for surface waves but unnecessarily restricts results for internal waves and precludes application to waves influenced by inertial or Coriolis forces.

A primary purpose of this paper is to generalize and extend previous work in order to remove restrictive assumptions imposed on sea water velocity fields. We presume only that the divergence of velocity vanishes and so only exclude flow fields produced by sound waves. By using Faraday's and Ampere's laws to derive a vector potential function for the induced electromagnetic field, we demonstrate that the electromagnetic field generated by a progressive ocean wave in a horizontally stratified ocean is a sum of a transverse electric type field, a transverse magnetic type field, and an electrostatic field. Velocity components in a vertical plane containing the direction of wave propagation generate the transverse electric part of the field, and the velocity component normal to the plane generates the transverse magnetic part of the field. The electrostatic part of the field results from surface charges that halt vertical electric currents at an ocean surface.

At wave frequencies appreciably greater than tidal frequencies, sea water velocities are elliptically polarized in a vertical plane containing the direction of wave propagation. Consequently,

surface waves and high frequency internal waves generate transverse electric type fields alone, accompanied by an electrostatic field. Both transverse electric and transverse magnetic type fields, together with accompanying electrostatic fields, are generated by ocean waves with frequencies of the order of tidal frequencies. Transverse magnetic type fields, however, vanish above an ocean surface and so are confined below the surface, but electrostatic and transverse electric type fields are present both above and below the surface.

A second purpose of this paper is to put forth and examine means of obtaining a measure of ocean wave spectra, for both surface and internal waves, by measuring noise spectra of magnetic field gradients above an ocean surface.* The magnetic field generated by moving sea water is a linear transformation of the sea water velocity field and so is effectively the response of a linear network — the Earth's magnetic field. Consequently, a correspondence between noise spectra of magnetic fields and, hence, magnetic field gradients induced by ocean waves and velocity spectra of corresponding wave fields is provided by a transfer function that is, in effect, the magnetic field induced by a progressive ocean wave of unit magnitude.

At present, measurements of magnetic fields over oceans are largely limited to mapping large scale irregular features of the Earth's field [Bullard and Mason, 1963, and Heirtzler, 1970] because of limitations on sensitivity and difficulties of separating spatial and temporal fluctuations produced by ocean waves, ionospheric sources, and geologic sources. Nonetheless, recent measurements of magnetic

*Measuring spatial gradients in magnetic fields rather than field magnitudes suppresses noise produced by ionospheric and other remote sources.

fields induced by surface waves using so-called total field or alkali vapor magnetometers on fixed and floating platforms are reported by Maclure, et al. [1964], Kozlov, et al. [1971], Kravtsov, et al. [1971], and Groskaya, et al. [1972]. Baker and Graefe, [1968] report limited success in measuring surface wave spectra using a rubidium vapor magnetometer towed by aircraft. The advent of superconducting magnetometers based on the Josephson effect [Clarke, 1974 and Goodman, et al. 1973], however, offers unprecedented sensitivities (10^{-4} gamma/$\sqrt{\text{hertz}}$) that effectively allow measurement of field gradients at a point with sensitivities of the order of 10^{-4} (gamma/m)/$\sqrt{\text{hertz}}$, with the definition one gamma equals 10^{-9} webers/m^2.

In order to examine potential utility of superconducting magnetic gradiometers for measuring ocean wave spectra, we first determine magnetic fields induced by progressive ocean waves and so specify transfer functions relating noise spectra of magnetic fields and corresponding ocean wave spectra. We find that magnetic fields induced above the surface by progressive waves are circularly polarized in a vertical plane containing the direction of wave propagation and that their magnitudes decrease exponentially with height above the surface as e^{-kh}, where k denotes wave number of a progressive wave. Magnitudes at the surface are proportional to a weighted average of sea water speed over an ocean depth. Magnetic field gradients induced at the surface by both surface and internal waves provide sensible signals for superconducting magnetic gradiometers provided surface speeds exceed 1 cm/sec or so.

In what follows, we first obtain a general solution for electromagnetic fields induced by progressive waves in a horizontally stratified ocean. We then restrict consideration to wave frequencies far above tidal frequencies and present solutions for transverse electric type fields induced by surface waves and by internal waves in an exponentially stratified ocean and an ocean stratified in two layers. We conclude by demonstrating that vertically spaced measurements of noise spectra of magnetic field gradients above an ocean surface offer a unique and promising means of obtaining a measure of surface and internal wave spectra for three reasons: (1) proportionality of magnetic field strengths above the surface to a weighted average of sea water speed over an ocean depth provides a response depending on the mode structure of internal waves, (2) exponential decrease of field strengths with height above the surface as e^{-kh} provides means of wave number discrimination, which together with frequency discrimination offers means of resolving internal wave spectra mode by mode, and (3) noise spectra of magnetic field gradients are effectively cross-spectra and so provide directional information on wave spectra from an effective point measurement.

II. ELECTROMAGNETIC FIELD GENERATED BY MOVING SEA WATER

Sea water moving across the Earth's magnetic field induces an electric current in the sea that generates an electromagnetic field characteristic of the velocity field in the sea. Motion of sea water with velocity $\vec{V}$ across the Earth's constant magnetic field, $\vec{B}$, produces a conduction current, $\vec{J}$, in the sea that we express in MKS units as

$$\vec{J} = \sigma(\vec{E} + \vec{V} \times \vec{B}) \quad , \tag{1}$$

where $\vec{E}$ is the electric field in a stationary frame of reference and $\vec{V} \times \vec{B}$ is the electric field induced by sea water motion. Electrical conductivity, σ, of sea water is about 4 mhos/m. At frequencies characteristic of sea water oscillations, displacement currents in the sea are negligibly small compared to conduction currents [Kraichman, 1970], so that we determine the magnetic field, $\vec{b}$, of the induced current by using Ampere's law in the form

$$\vec{\nabla} \times \vec{b} = \mu_0 \vec{J} \quad , \tag{2}$$

where $\mu_0 = 4\pi \times 10^{-7}$ henry/m. A second order contribution to the induced current proportional to $\vec{V} \times \vec{b}$ is negligible, since amplitudes of induced magnetic fields are several orders of magnitude smaller than the Earth's field. Finally, electric and magnetic fields are

related by Faraday's law, so that

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{b}}{\partial t} \quad . \tag{3}$$

Equations (1) (2), and (3) together with appropriate boundary conditions determine the electromagnetic field, $\vec{E}$ and $\vec{b}$, generated by sea water motion having a velocity field $\vec{V}$. Because electrical conductivity of sea water is small, the Lorentz body force, $\vec{J} \times \vec{B}$, acting on the fluid as a result on the induced current is negligibly small compared to pressure and buoyancy forces. Consequently, hydromagnetic coupling effects are absent.

We suppose that sea water flow fields are described by motion of an incompressible fluid and so exclude flow fields produced by sound waves. The divergence of velocity then vanishes, and the velocity field is the curl of a vector stream function, $\vec{\Psi}$; namely,

$$\vec{V} = \vec{\nabla} \times \vec{\Psi} \quad , \tag{4a}$$

and we choose the gauge

$$\vec{\nabla} \cdot \vec{\Psi} = 0.$$

The electric field induced by moving sea water, then, is the sum,

$$\vec{V} \times \vec{B} = \vec{\nabla} \times (\vec{\Psi} \times \vec{B}) - \vec{\nabla}(\vec{\Psi} \cdot \vec{B}) \quad , \tag{4b}$$

of transverse and longitudinal type fields.

A. VECTOR POTENTIAL FORMULATION

In order to express the electromagnetic field in terms of the vector stream function, we introduce a vector potential, $\vec{A}$, for the induced magnetic field, so that

$$\vec{b} = \vec{\nabla} X \vec{A} \quad . \tag{5a}$$

By using Faraday's law, we express the electric field as

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} - \vec{\nabla}\phi \quad , \tag{5b}$$

where ϕ is a scalar potential function. Because the divergence of $\vec{J}$ vanishes, vector and scalar potentials are related by the expression

$$\nabla^2\phi + \frac{\partial \vec{\nabla} \cdot \vec{A}}{\partial t} = -\nabla^2 (\vec{\Psi} \cdot \vec{B}) \quad . \tag{5c}$$

From Ampere's law, we obtain the additional relation

$$\vec{\nabla} X \vec{\nabla} X \vec{A} + \mu_o \sigma \frac{\partial \vec{A}}{\partial t} = \mu_o \sigma \vec{\nabla} X (\vec{\Psi} X \vec{B})$$

$$- \mu_o \sigma \vec{\nabla} (\phi + \vec{\Psi} \cdot \vec{B}) \tag{5d}$$

between potentials.

By specifying the gauge

$$\vec{\nabla} \cdot \vec{A} = -\mu_o \sigma \chi \quad , \tag{6a}$$

where

$$\chi = \phi + \vec{\Psi} \cdot \vec{B} \quad , \tag{6b}$$

we obtain the expressions

$$\nabla^2 \chi - \mu_o \sigma \frac{\partial \chi}{\partial t} = 0 \tag{7a}$$

and

$$\nabla^2 \vec{A} - \mu_o \sigma \frac{\partial \vec{A}}{\partial t} = -\mu_o \sigma \vec{\nabla} \times (\vec{\Psi} \times \vec{B}) \tag{7b}$$

from Equations (5c) and (5d), respectively.

Because Equation (7a) is homogeneous, however, we make a gauge transformation to a gauge in which $\vec{\nabla} \cdot \vec{A}$ and χ vanish and express the electric field and current in terms of the vector stream function and a vector potential alone as

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} + \vec{\nabla}(\vec{\Psi} \cdot \vec{B}) \tag{8a}$$

and

$$\vec{J} = -\sigma \frac{\partial \vec{A}}{\partial t} + \sigma \vec{\nabla} \times (\vec{\Psi} \times \vec{B}) \quad , \tag{8b}$$

where the vector potential $\vec{A}$ is a solenoidal solution of Equation (7b).

Above the ocean surface, both the stream function and electrical conductivity vanish, so the corresponding vector potential, $\vec{A}_v$, is a solenoidal solution of the vector form of Laplace's equation. Below the ocean floor, the stream function vanishes, and the vector potential, $\vec{A}_s$, is a solenoidal solution of the homogeneous form of

Equation (7b) with an electrical conductivity σ_s corresponding to that of suboceanic strata. The electric field and current above and below the ocean are given by corresponding forms of Equations (8a) and (8b), respectively. Finally, electric and magnetic fields produced by moving sea water vanish far above and below the ocean, and their horizontal components are continuous across the surface and floor of the ocean.

B. SCALAR POTENTIAL COMPONENTS

In order to determine the electromagnetic field generated by a specified velocity field, then, we determine the vector potential generated by the corresponding vector stream function, so that derived electric and magnetic fields vanish far above and below the ocean and have continuous horizontal components. Because both the vector stream function and vector potential are solenoidal, each is determined by two scalar potential components [Morse and Feshbach, 1953]. For our purposes, we represent oceans as horizontally stratified and so express $\vec{\Psi}$ and $\vec{A}$ in terms of scalar potential components Ψ_1, Ψ_2 and A_1, A_2, respectively, as

$$\vec{\Psi} = \vec{\nabla} \times (\Psi_1 \hat{z} + \hat{z} \times \vec{\nabla} \Psi_2) \tag{9a}$$

and

$$\vec{A} = \vec{\nabla} \times (A_1 \hat{z} + \hat{z} \times \vec{\nabla} A_2) \quad , \tag{9b}$$

where $\hat{z}$ is a unit vector directed vertically downward.

From Equation (7b), we then find that each pair of scalar potentials, A_1, Ψ_1 and A_2, Ψ_2, is related by the expression

$$\nabla^2 A_j - \mu_o \sigma \frac{\partial A_j}{\partial t} = -\mu_o \sigma \vec{B} \cdot \vec{\nabla} \Psi_j \quad . \tag{10}$$

Furthermore, each scalar component, A_{v1} and A_{v2}, of the vector potential above the ocean surface is a solution of Laplace's equation, and each scalar component, A_{s1} and A_{s2}, of the vector potential below the ocean floor is a solution of the homogeneous form of Equation (10). Because scalar potential components of the vector stream function and vector potential are paired, each component of the vector stream function excites its own mode of electromagnetic field.

C. ELECTROMAGNETIC FIELD GENERATED BY A PROGRESSIVE WAVE

In order to specify the mode structure of electromagnetic fields generated by ocean waves, we express each of the scalar potential components as a Fourier integral of a potential function corresponding to a horizontally progressing ocean wave having an angular frequency ω and horizontal wave vector $\vec{k}$. For example, we express scalar potential components of the vector stream function as

$$\Psi_j(z, \vec{r}, t) = \iint d\omega d\vec{k}\, \psi_j(z, \vec{k}, \omega)\, e^{i(\omega t - \vec{k} \cdot \vec{r})} \quad , \tag{11a}$$

where $\vec{r}$ is a horizontal coordinate vector. The Fourier coefficient $\psi_j(z, \vec{k}, \omega)$ is the vertical profile of a wave progressing horizontally in a direction $\vec{k}$ with an angular frequency ω.

Each vector field, then, is a Fourier integral of a corresponding vector profile. We express the vector stream function, for example, as

$$\vec{\Psi}(z,\vec{r},t) = \iint d\, d\vec{k}\, \vec{\Psi}(z,\vec{k},\omega)\, e^{i(\omega t - \vec{k}\cdot\vec{r})} \quad . \tag{11b}$$

By using Equations (9a) and (11a), we express the vector profile $\vec{\psi}(z,\vec{k},\omega)$ in terms of corresponding potential function profiles ψ_1 and ψ_2 as

$$\vec{\psi} = ik(\psi_1 \hat{n} + \hat{n} X \vec{\nabla}_r \psi_2) \quad . \tag{11c}$$

The reduced gradient operator, $\vec{\nabla}_r$, is defined by

$$\vec{\nabla}_r \equiv \hat{z}\frac{d}{dz} - ik\hat{k} \quad , \tag{11d}$$

and $\hat{n} = \hat{z} X \hat{k}$, where $\hat{k}$ is a horizontal unit vector pointing in the direction of wave propagation. By taking the curl of Equation (11b), then, we obtain the expression

$$\vec{\nu} = ik(\hat{n}\nabla_r^2 \psi_2 - \hat{n} X \vec{\nabla}_r \psi_1) \tag{11e}$$

for the vector profile, $\vec{\nu}(z,\vec{k},\omega)$, of the velocity field.

Similarly, vector profiles, $\vec{a}$ and $\vec{\beta}$, of the vector potential and magnetic field, respectively, are expressed as

$$\vec{a} = ik(a_1 \hat{n} + \hat{n} X \vec{\nabla}_r a_2) \tag{12a}$$

and

$$\vec{\beta} = ik(\hat{n}\nabla_r^2 a_2 - \hat{n} X \vec{\nabla}_r a_1) \quad . \tag{12b}$$

Vector profiles of electric field and current, $\vec{\epsilon}$ and $\vec{j}$, are given by

$$\vec{\epsilon} = \omega k\,(a_1\hat{n} + \hat{n}\,X\vec{\nabla}_r a_2) + \vec{\nabla}_r(\vec{\Psi}\cdot\vec{B}) \tag{12c}$$

and

$$\mu_0\vec{j} = -ik[\hat{n}\nabla_r^2 a_1 + \hat{n}\,X\,\vec{\nabla}_r(\nabla_r^2 a_2)] \quad . \tag{12d}$$

Finally, we find from Equation (10) that vertical profiles of scalar components of the vector stream function and vector potential are paired according to the relation

$$\nabla_r^2 a_j - i\mu_0\sigma\omega a_j = -\mu_0\sigma\vec{B}\cdot\vec{\nabla}_r\psi_j \quad . \tag{13}$$

Above the surface, vertical profiles, a_{v1} and a_{v2}, are solutions of Laplace's equation with a reduced form of the Laplacian operator. Vertical profiles in suboceanic strata, a_{s1} and a_{s2}, are solutions of the homogeneous form of Equation (13).

As is evident from Equations (12a), (12b), and (12c), the scalar component a_1 of the vector potential, and hence the scalar component ψ_1 of the vector stream function, generates a transverse electric type field, so called because the corresponding electric field vector is normal to the plane containing the magnetic field and wave vectors. The scalar component a_2 of the vector potential, and hence the scalar component ψ_2 of the vector stream function, generates a so-called transverse magnetic type field. In each case, the electric field also contains an electrostatic type field generated by the gradient of $\vec{\Psi}\cdot\vec{B}$. The electrostatic field vector lies in a vertical plane containing the wave vector.

We observe from Equation (12b) that the transverse magnetic mode vanishes above the surface, because scalar components of the vector potential are harmonic above the surface. Although both modes propagate below the surface, the transverse electric mode alone propagates above the surface. From Equation (12d), we also note that the transverse magnetic mode is excited by electric currents in a vertical plane containing the wave vector and that the transverse electric mode is excited by horizontal electric currents normal to the plane.

By imposing continuity of horizontal components of electric and magnetic fields at the ocean surface and floor, we find that modes are not coupled at boundaries, so that the mode structure of electromagnetic fields generated by ocean waves is determined by the vector stream function. If the vector stream function contains either the scalar component ψ_1 alone or the scalar component ψ_2 alone, then the electromagnetic field is the sum of a transverse electric type field and an electrostatic type field, in the first case, and is the sum of a transverse magnetic type field and an electrostatic type field, in the second case. In each case, the electrostatic part of the field results from surface charge required to halt vertical electric currents at the surface. For vector stream functions containing both scalar components, the electromagnetic field is a sum of transverse electric and transverse magnetic type fields below the surface and a transverse electric type field above the surface, along with corresponding electrostatic type fields.

We observe from Equation (11e) that the scalar component ψ_2 of the vector stream function produces a velocity component normal to a vertical plane containing the wave vector. Consequently, velocity fields of surface and internal waves at frequencies appreciably greater than tidal frequencies, for which the velocity vector is elliptically

polarized in a vertical plane containing the wave vector [Phillips, 1969], are described by the scalar component ψ_1 alone. Velocity fields of waves with frequencies of the order of tidal frequencies contain a transverse component of velocity and so generate a transverse magnetic mode as well as a transverse electric mode. The transverse magnetic mode, however, is confined below the surface.

Because we are interested herein in internal and surface waves with frequencies appreciably greater than tidal frequencies, we confine further consideration to transverse electric type fields.

III. TRANSVERSE ELECTRIC TYPE FIELDS GENERATED BY PROGRESSIVE WAVES

Progressive waves having velocity vectors elliptically polarized in a vertical plane containing the wave vector generate transverse electric type fields. The scalar component ψ_2 of the vector stream function then vanishes, and by using Equation (11e), we express the vector profile of the velocity field in terms of circularly polarized components as

$$\vec{\nu} = \frac{\nu_z}{\sqrt{2}} \left[(1+p)\hat{\nu} + (1-p)\hat{\nu}^*\right] \quad , \tag{14a}$$

where the circularly polarized unit vector $\hat{\nu}$ is defined by

$$\hat{\nu} = \frac{1}{\sqrt{2}} (\hat{z} + i\hat{k}) \quad , \tag{14b}$$

$\hat{\nu}^*$ is the complex conjugate of $\hat{\nu}$, and the polarization coefficient, $p(z,\vec{k},\omega)$ is defined by

$$p = -\frac{1}{k} \frac{d \, \ell n \, \nu_z}{dz} \quad . \tag{14c}$$

The vertical component, ν_z, of the vector profile is related to the scalar component ψ_1 of the vector stream function by the expression

$$\nu_z = k^2\psi_1 \quad . \tag{14d}$$

The unit vector $\hat{\nu}$ has the convenient property that $\hat{\nu} \times \hat{n} = i\hat{\nu}$. Circular polarization of $\hat{\nu}$ is left-handed with respect to $\hat{n}$, and that of $\hat{\nu}^*$ is right-handed.

Vector profiles of corresponding transverse electric type fields are given by Equations (12b) and (12c) with the scalar component a_2 of the vector potential profile equal to zero. The remaining scalar component, a_1, of the vector potential profile is the solution of Equation (13) that gives continuous horizontal components of electric and magnetic fields.

Scalar components of the vector potential above the surface are harmonic, and fields vanish far above the surface. Consequently, we express vector profiles of electric and magnetic fields above the surface, $\vec{\epsilon}_v$ and $\vec{\beta}_v$, in terms of the complex amplitude of the magnetic field vector profile at the surface, β_o, as

$$\vec{\epsilon}_v = \beta_o\left(\frac{\omega}{k}\right)\frac{e^{kz}}{\sqrt{2}}\,\hat{n} + i\nu_{zo}(\vec{B}\cdot\hat{n})\,e^{kz}\sqrt{2}\,\hat{\nu}^* \tag{15a}$$

and

$$\vec{\beta}_v = \beta_o\,e^{kz}\,\hat{\nu}^* \quad , \tag{15b}$$

where $z \leqslant 0$ above the surface and ν_{zo} is the vertical component of the velocity profile at the surface. We note that the magnetic field above the surface is circularly polarized with a right-handed sense of rotation with respect to the unit vector $\hat{n}$.

Information on suboceanic electrical conductivity profiles is sparse. Conductivity of the Earth's crust below the oceans is much less than that of sea water. Conductivity increases with depth within the Earth's mantle below the crust and becomes much greater than that of sea water. Estimates of the extent of the poorly conducting crustal layer beneath the oceans range from a few tens of kilometers to a few hundreds of kilometers [Cox, et al., 1970], which is of the order of ionospheric heights. For simplicity, then, we represent the poorly conducting layer as a vacuum and consider ocean wavelengths that are much less than the extent of the poorly conducting layer and so approximate conductivity profiles of suboceanic strata as a vacuum region of indefinite extent below the ocean floor.

Consequently, scalar components of the vector potential are harmonic below the ocean floor as well, and we express suboceanic vector profiles of electric and magnetic fields, $\vec{\epsilon}_S$ and $\vec{\beta}_S$, in terms of the complex amplitude of the magnetic field vector profile at the bottom, β_D, as

$$\vec{\epsilon}_v = \beta_D\left(\frac{\omega}{k}\right)\frac{e^{-k(z-D)}}{\sqrt{2}}\,\hat{n} \tag{16a}$$

and

$$\vec{\beta}_v = \beta_D\, e^{-k(z-D)}\,\hat{\nu} \;, \tag{16b}$$

where $z \geqslant D$ and D is the ocean depth. We note that the electrostatic part of the electric field vanishes in suboceanic strata, because the vertical component of velocity vanishes at the ocean floor.

By requiring continuity of horizontal components of electric and magnetic fields, we find that the logarithmic derivative of the

scalar component a_1, namely $(da_1/dz)/a_1$, equals k at the ocean surface and -k at the ocean floor. By using the Green's function

$$g(z,\zeta) = -\frac{1}{2k} e^{-k|z-\zeta|} , \tag{17a}$$

we write Equation (13) for the scalar component a_1 as the integral equation

$$a_1(\zeta) = \delta\int_0^D \vec{B}\cdot\vec{\nabla}_r\psi_1(z)e^{-k|z-\zeta|}dz$$

$$-\delta\left(\frac{i\omega}{k}\right)\int_0^D a_1(z)e^{-k|z-\zeta|}kdz , \tag{17b}$$

where

$$\delta = \frac{\mu_0\sigma}{2k} .$$

Expression (17b) is a Fredholm integral equation of the second kind.*

We write the solution for a_1 as the Neuman series [Tricomi, 1957]

$$a_1 = \delta Gf + \sum_{m=1}^{\infty}\left(\frac{-i\omega}{k}\right)^m \delta^{m+1} G^{m+1} f \tag{18a}$$

where

$$kf = \vec{B}\cdot\vec{\nabla}_r\psi_1$$

* The trivial solution, $a_1=0$, is the only solution of the homogeneous form of Equation (17b), because eigenvalues of a symmetric kernel are real [Tricomi, 1957].

and the integral operator G is defined by

$$Gf = \int_0^D f(z) e^{-k|z-\zeta|} kdz \quad . \tag{18b}$$

The quantity δ, whose reciprocal is a measure of the speed at which fields diffuse over a distance of one ocean wavelength, is given in terms of ocean wavelength, λ in meters, by

$$\delta = (4 \times 10^{-7})\lambda \quad \text{sec/m} \tag{18c}$$

and so is small for wavelengths of interest herein. Consequently, we approximate a_1 to first order in δ by retaining only the first term in Equation (18a), which is an integral over the source.

From Equations (12b), (12c), and (12d), we then obtain the first order expressions

$$\vec{\epsilon}(\zeta) = -i\delta\left(\frac{\omega}{k}\right)\hat{n}\int_0^D \epsilon_{i,n}(z) e^{-k|z-\zeta|}\, kdz - (\vec{B}\cdot\hat{n})(\vec{\nu}X\hat{n}) \tag{19a}$$

and

$$\vec{\beta}(\zeta) = -i\delta\sqrt{2}\,[\hat{\nu}\int_0^{\zeta} \epsilon_{i,n}(z) e^{-k(\zeta-z)} kdz$$

$$+ \hat{\nu}^* \int_{\zeta}^D \epsilon_{i,n}(z) e^{-k(z-\zeta)} kdz] \quad , \tag{19b}$$

for vector profiles of electric and magnetic fields in the ocean $(0 \le \zeta \le D)$, and the first order expression

$$\vec{j} = \sigma\epsilon_{i,n}\hat{n} \quad , \tag{19c}$$

for electric current in the ocean. The profile of the transverse component of the electric field induced by sea water motion, $\epsilon_{i,n}(z)$, is given by

$$\epsilon_{i,n} = \hat{n}\cdot(\vec{v} \times \vec{B}) \quad . \tag{19d}$$

From Equations (19a) and (19c), we note that

$$\vec{j} - \sigma\vec{\epsilon} = \sigma(\vec{v} \times \vec{B}) + i\delta(\tfrac{\omega}{k})\hat{n} \int_0^D \epsilon_{i,n}(z) e^{-k|z-\zeta|} k dz \quad . \tag{20}$$

For consistency, then, we require the product $\delta(\omega/k)$ to be small as well. Phase speeds of ocean waves are less than $\sqrt{gD}$ or about 300 m/sec in the deepest ocean, where $D \sim 10$ km, so that a first order approximation is appropriate for ocean wavelengths less than several kilometers, which is quite adequate for our purposes.*

We conclude, then, that the electric field component induced by magnetic field oscillations, which is normal to the plane of polarization of velocity, does not appreciably influence electric current, so that electric current, in effect, is driven by the transverse component of the electric field induced by sea water motion. Consequently, electric current and magnetic field are determined directly by the velocity field. Magnitude of the electric current is proportional to local sea water speed. Magnetic field strength is proportional to a weighted average of the profile of sea water speed. The component of the electric field induced by sea water motion that is in the plane of polarization of velocity simply produces surface charges, which halt vertical electric currents.

* Although obtaining exact solutions for the scalar component a_1, and hence for induced fields, is straightforward, resulting expressions are cumbersome and obscure.

A. VELOCITY PROFILES

In a stratified ocean having a Brunt-Väisälä frequency profile, N(z), the vertical component of the vector profile of velocity of a progressive wave is a solution of the integral equation

$$\nu_z(z) = \frac{1}{\omega^2} \int_0^D N^2(\zeta)\nu_z(\zeta)G(\zeta, z)k\,d\zeta$$

$$+ \nu_{zo} \frac{\sinh k(D-z)}{\sinh kD} \quad , \tag{21a}$$

which is equivalent to the differential formulation given by Phillips, [1969]. The symmetric kernel $G(\zeta, z)$ is given by

$$G(\zeta, z) = \frac{\sinh k\zeta \sinh k(D-z)}{\sinh kD} \; , \text{ for } \zeta \leq z \quad , \tag{21b}$$

and by

$$G(\zeta, z) = \frac{\sinh kz \sinh k(D-\zeta)}{\sinh kD} \; , \text{ for } \zeta \geq z \quad . \tag{21c}$$

The horizontal component of the vector profile of velocity, $\nu_k(z)$, is determined by the relation

$$i\nu_k = \frac{1}{k} \frac{d\nu_z}{dz} \quad , \tag{22a}$$

since the divergence of velocity vanishes. Vertical and horizontal components at the surface, ν_{zo} and ν_{ko} respectively, are related by the expression

$$i\nu_{zo} = \left(\frac{\omega^2}{gk}\right) \nu_{ko} \quad , \tag{22b}$$

where $g = 9.8\ m/sec^2$, and the vertical component vanishes at the ocean bottom. The dispersion relation for progressive waves is given by the expression

$$i\nu_{ko}\left(\tanh kD - \frac{\omega^2}{gk}\right)\cosh kD = \frac{1}{\omega^2}\int_0^D N^2(\zeta)\,\nu_z(\zeta)\sinh k(D-\zeta)k\,d\zeta\,. \quad (23)$$

Equation (21a) for the vertical component of the vector profiles of velocity of a progressive wave is a Fredholm integral equation of the second kind, so that properties of solutions are well known [Tricomi, 1957]. Eigenfunctions of the homogeneous form of Equation (21a) correspond to internal wave modes with associated eigenfrequencies ω_n. At internal wave frequencies, the ratio ω_n^2/gk is negligibly small, so that surface displacements effectively vanish. Profiles of the vertical component of velocity are then a superposition of eigenfunctions or modes. At frequencies much greater than internal wave frequencies, the profile effectively is that obtained from Equation (21a) for a vanishing Brunt-Väisälä frequency, which is the profile corresponding to surface waves on a uniform ocean.

For Brunt-Väisälä frequency profiles corresponding to actual ocean stratification, solutions ordinarily must be obtained numerically or by approximate methods such as a WKB approximation, which is appropriate for slowly varying profiles. For purposes of subsequent discussion, however, we consider three model profiles that represent three ideal types of ocean stratification: (1) a uniform ocean, (2) an exponentially stratified ocean, and (3) an ocean stratified in two layers.

The Brunt-Väisälä frequency vanishes for a uniform ocean, and surface waves are then the only mode of progressive wave motion.

The corresponding profile of the vertical component of velocity and dispersion relation are immediately evident from Equations (21a) and (23), respectively.

The frequency profile is constant or uniform for an exponentially stratified ocean. We find that the profile of the vertical component of velocity for an internal wave is then given by the eigenfunction or modal expansion

$$\nu_z(z) = -i\sum_{n=1} \nu_{ko,n}(-1)^n \frac{kD}{n\pi} \sin n\pi(1-z/D) \tag{24a}$$

and that

$$\nu_{ko} = \sum_{n=1}^{\infty} \nu_{ko,n} \quad , \tag{24b}$$

so that $\nu_{ko,n}$ is the horizontal component of velocity at the surface resulting from the nth mode. Corresponding dispersion relations are given by

$$\omega_n^2 = \frac{N^2}{1 + \left(\frac{n\pi}{kD}\right)^2} \quad , \quad n = 1, 2, 3, \ldots \tag{24c}$$

For a two-layer stratification, we represent the frequency profile in terms of a delta function, $\delta(z-d)$, centered at a depth d, so that

$$N^2(z) = g\left(\frac{\Delta\rho}{\rho}\right)\delta(z-d) \quad , \tag{25a}$$

where $\Delta\rho/\rho$ is the fractional increase in density between layers, which is of the order of 10^{-3} in the oceans. By neglecting terms of the order of ω^2/gk, we then obtain from Equation (21a) the profile

$$\nu_z(z) = i\,\nu_{ko}\left[\frac{\sinh kD}{\sinh k(D-d)}\right] G(d, z) \tag{25b}$$

and, from Equation (23), the dispersion relation

$$\omega^2 = gk\left(\frac{\Delta\rho}{\rho}\right)\left[\frac{\sinh kd \sinh k(D-d)}{\sinh kD}\right] , \tag{25c}$$

for a two-layer stratification representing a sharp thermocline at a depth d. We note that the velocity profile is described by a single mode, as for surface waves on a uniform ocean.

B. ELECTROMAGNETIC FIELD PROFILES

The profile of the transverse component of the electric field induced by sea water motion, as given by Equation (19d), is proportional to the horizontal component of velocity in polar regions and to the vertical component of velocity in equatorial regions. As a result, magnetic fields are generated by the horizontal component of velocity in polar regions and by the vertical component of velocity in equatorial regions. At intermediate latitudes, however, both components of velocity contribute, and fields are a superposition of fields corresponding to polar and equatorial regions with relative proportions depending on magnetic latitude.

Nonetheless, the horizontal component of the vector profile of velocity is proportional to the vertical gradient of the vertical component, because the divergence of velocity vanishes, so that by using Equations (14a) and (19b) and integration by parts, we express the vector profile of the induced magnetic field in terms of the vertical component, ν_z, alone as

$$\vec{\beta}(z) = \delta\left\{\sqrt{2}(\vec{B}\cdot\hat{z})\left[\nu_z(z) - \nu_{zo}e^{-kz}\right] - 2(\vec{B}\cdot\hat{\nu})\int_0^z \nu_z(\zeta)e^{-k(z-\zeta)}kd\zeta\right\}\hat{\nu}$$

$$-\delta\left\{\sqrt{2}(\vec{B}\cdot\hat{z})\,\nu_z(z) - 2(\vec{B}\cdot\hat{\nu}^*)\int_z^D \nu_z(\zeta)e^{-k(\zeta-z)}kd\zeta\right\}\hat{\nu}^* \;, \quad (26a)$$

within the sea ($0 \leq z \leq D$), and as

$$\vec{\beta}_v(h) = -\delta e^{-kh}\left[\sqrt{2}(\vec{B}\cdot\hat{z})\nu_{zo} - 2(\vec{B}\cdot\hat{\nu}^*)\int_0^D \nu_z(\zeta)e^{-k\zeta}kd\zeta\right]\hat{\nu}^* \quad (26b)$$

at a height h above the surface. We again note that the magnetic field above the surface is circularly polarized and that its magnitude decreases exponentially with height as e^{-kh}. Magnetic field strengths at the surface depend on a weighted integral of the vertical component of velocity over an ocean depth that is a Laplace transform of the vertical component of velocity. Because of the factor $e^{-k\zeta}$ in the integral, the contribution of deep eddy currents to magnetic field strengths at the surface is greater at smaller wave numbers. Finally, we observe from Equation (26b) that the gradient in a direction $\hat{u}$ of the vector profile of the magnetic field induced above the surface is simply expressed by

$$(\hat{u}\cdot\vec{\nabla}_r)\,\vec{\beta}_v = k\,\sqrt{2}\,(\hat{u}\cdot\hat{\nu}^*)\,\vec{\beta}_v \quad . \quad (26c)$$

From Equations (19a) and (19b), we find that corresponding vector profiles of electric fields are given by

$$\vec{\epsilon} = \left(\frac{\omega}{k}\right)(\hat{z}\cdot\vec{\beta}_v)\hat{n} - i(\vec{B}\cdot\hat{n})[(\vec{\nu}\cdot\hat{\nu}^*)\hat{\nu} - (\vec{\nu}\cdot\hat{\nu})\hat{\nu}^*] \quad (26d)$$

within the sea and, from Equations (15a) and (15b), by

$$\vec{\epsilon}_v = \left(\frac{\omega}{k}\right)(\hat{z}\cdot\vec{\beta}_v)\hat{n} + i\nu_{zo}(\vec{B}\cdot\hat{n})\,{}^{-kh}\sqrt{2}\,\hat{\nu}^* \quad (26e)$$

above the surface. We note that the impedance of the transverse electric part of the field is proportional to ocean wave phase speeds. The electrostatic part of the field vanishes in polar regions and is circularly polarized above the surface with a magnitude proportional to the vertical component of velocity at the surface.

Because velocity profiles of progressive waves in a stratified ocean depend on the Brunt-Väisälä frequency profile, description of electromagnetic fields generated by progressive waves requires a model for the frequency profile. In what follows, we first present results for electromagnetic fields generated by surface waves on a uniform ocean and then for electromagnetic fields generated by internal waves in an exponentially stratified ocean and an ocean stratified in two layers.

IV. ELECTROMAGNETIC FIELDS GENERATED BY SURFACE WAVES

By evaluating integrals in Equations (26a) and (26b) for velocity profiles corresponding to surface waves on a uniform ocean of depth D, we find that vector profiles of magnetic fields generated by surface waves are given by

$$\vec{\beta} = \frac{-\mu_o \sigma \nu_{zo}}{2k \sinh kD} \left\{ \left[(\vec{B}\cdot\hat{\nu}^*) e^{-kD} \sinh kz + (\vec{B}\cdot\hat{\nu}) kz e^{k(D-z)} \right] \hat{\nu} \right.$$

$$\left. + \left[(\vec{B}\cdot\hat{\nu}) \sinh k(D-z) + (\vec{B}\cdot\hat{\nu}^*) k(D-z) e^{-k(D-z)} \right] \hat{\nu}^* \right\} \quad (27a)$$

within the sea $(0 \leq z \leq D)$ and by

$$\vec{\beta}_v = \frac{-\mu_o \sigma}{2k} \nu_{zo} \left[(\vec{B}\cdot\hat{\nu}) + \frac{kDe^{-kD}}{\sinh kD} (\vec{B}\cdot\hat{\nu}^*) \right] e^{-kh} \hat{\nu}^* \quad (27b)$$

at a height h above the surface. Corresponding vector profiles of electric fields are given by

$$\vec{\epsilon} = \left(\frac{\omega}{k}\right) (\hat{z}\cdot\vec{\beta})\hat{n} - \frac{i\nu_{zo}}{\sinh kD} \frac{(\vec{B}\cdot\hat{n})}{\sqrt{2}} \left[\hat{\nu} e^{k(D-z)} + \hat{\nu}^* e^{-k(D-z)} \right] \quad (27c)$$

within the sea and by

$$\vec{\epsilon}_v = \left(\frac{\omega}{k}\right) (\hat{z}\cdot\vec{\beta}_v)\hat{n} + i\nu_{zo} (\vec{B}\cdot\hat{n}) e^{-kh} \sqrt{2}\, \hat{\nu}^* \quad (27d)$$

above the surface.

We observe from Equations (27a) and (27c) that the strength of electric and magnetic fields generated by surface waves is proportional to the vertical component of sea water velocity at the surface. For small wave amplitudes, then, field strengths are also proportional to wave height, ξ_0, because of the relation

$$\nu_{zo} = i\xi_0 \sqrt{gk \tanh kD} \quad , \tag{28a}$$

which expresses the kinematic relation between vertical displacement and the vertical component of velocity at the surface, or to the horizontal component of sea water velocity at the surface, because of the relation

$$\nu_{zo} = -i\nu_{ko} \tanh kD \quad , \tag{28b}$$

which expresses the relation between velocity components at the surface.

For purposes of illustration, we choose an ocean depth of 1 km and take the value 6.24 X 10^4 gamma [Chapman, 1962] for the Earth's field strength in polar regions, B_p, with the definition one gamma equals 10^{-9} webers/m^2. We express the Earth's magnetic field in terms of the unit vectors $\hat{z}$, $\hat{k}$, and $\hat{n}$ as

$$\vec{B} = \frac{B_p}{2} \sqrt{1 + 3 \sin^2 \Phi} \; [\hat{z} \sin \phi + (\hat{k} \cos \theta - \hat{n} \sin \theta) \cos \phi] \, , \tag{29}$$

where θ is the angle between the direction of wave propagation and magnetic north, Φ denotes magnetic latitude, and the expression $\tan \phi = 2 \tan \Phi$ relates dip angle, ϕ, and magnetic latitude. Because wave height is the parameter commonly used to describe surface waves,

we present results in terms of field strengths generated per unit wave height and, for brevity, consider the magnetic field alone.

First, however, we observe from Equation (27b) that magnitudes of magnetic fields generated at the surface by surface waves on a deep ocean, $kD \gg 1$, are inversely proportional to wave number for a given surface speed. As shown by the solid curves in Figure 1, magnetic field strength at the surface increases with decreasing wave number for a constant surface speed, because the contribution of deep eddy currents to field strength at the surface is greater at smaller wave numbers. In polar regions, electric fields induced by sea water motion are proportional to the horizontal component of velocity, and magnetic field strengths at the surface per unit horizontal surface speed attain values of a few gamma/(cm/sec) at wavelengths much greater than an ocean depth. In equatorial regions, however, electric fields induced by sea water motion are proportional to the vertical component of velocity, and magnetic field strengths at the surface per unit vertical surface speed attain values of several tenths of a gamma/(cm/sec) at wavelengths much greater than an ocean depth. Maximum magnetic field strengths at the surface, per unit surface speed, are proportional to ocean depth and so attain values of the order of 10 gamma/(cm/sec) in the deepest oceans, for which $D \sim 10$ km.

Horizontal gradients in the direction of wave propagation of the magnetic field at the surface (which are simply expressed as $-ik\vec{\beta}_{vo}$), however, depend only on the product of wavenumber and ocean depth for a given surface speed. As shown by the dashed curves in Figure 1, magnitudes of horizontal gradients attain maximum values of the order of 10^{-3} (gamma/m)/(cm/sec) at wavelengths less than an ocean depth and decrease in proportion to kD at wavelengths much greater than an ocean depth.

Magnitudes of magnetic fields generated at the surface by a one meter high surface wave propagating in polar, equatorial, and intermediate latitudes ($\Phi = 90^{o}$, 0^{o}, and 45^{o}) are shown as functions of kD by the solid curves in Figure 2 and correspond to the left-hand coordinate scale, which is marked in units of gamma/m. Dashed curves delineate corresponding magnitudes of horizontal gradients in magnetic field strength along the direction of wave propagation and correspond to the right-hand coordinate scale, which is marked in units of (gamma/m)/m. The upper abscissa gives the ratio of ocean depth to wavelength, λ, of surface waves. Curves corresponding to equatorial and intermediate latitudes are drawn for wave propagation along magnetic meridians, $\theta = 0$ or π. Induced magnetic fields are independent of the direction of wave propagation in polar regions and are proportional to $\cos \theta$ in equatorial regions. At intermediate latitudes, the dependence on direction of wave propagation is only significant in a deep ocean.

As is evident in Figure 2, magnetic field strengths induced at the surface by a one meter high surface wave increase with decreasing wave number and attain maximum values of the order of 10 gamma/m in polar and intermediate latitudes at wavelengths much greater than an ocean depth. In equatorial regions, however, field strengths decrease with decreasing wave number for wavelengths much greater than an ocean depth, because the vertical component of sea water velocity at the surface decreases in proportion to wave number for a constant wave height, as is evident from Equation (28a). Horizontal gradients in magnetic field strength at the surface increase in proportional to $\sqrt{kD}$ for wavelengths less than an ocean depth and decrease in proportion to kD in polar regions and $(kD)^2$ in equatorial regions for wavelengths much greater than an ocean depth. Finally, magnetic field

strengths per unit wave height shown in Figure 2 scale according to the square root of ocean depth in kilometers, but magnitudes of horizontal gradients per unit wave height scal^ according to the inverse of the square root of ocean depth in kilometers.

In a deep ocean or for ocean wavelengths less than an ocean depth, we find from Equations (27a) and (28a) that vector profiles of magnetic fields generated by surface waves are given in terms of wave height by the expression

$$\vec{\beta} = \frac{-i\mu_o \sigma \xi_o}{2} \sqrt{\frac{g}{k}} \, (\vec{B}\cdot\nu) \, e^{-kz} \, (\hat{\nu}^* + 2kz\hat{\nu}) \tag{30a}$$

within the sea ($z \geq 0$) and by the expression

$$\vec{\beta}_v = \frac{-i\mu_o \sigma \xi_o}{2} \sqrt{\frac{g}{k}} \, (\vec{B}\cdot\hat{\nu}) \, e^{-kh} \, \hat{\nu}^* \tag{30b}$$

at a height h above the surface.* We observe that variations in polarization and magnetic field strength with depth are independent of latitude and depend only on the ratio of depth to ocean wavelength.

Profiles of polarization and magnitude of the magnetic field induced in a deep ocean by a one meter high surface wave propagating in polar regions with a wavelength of 10 m are shown in Figure 3, in which altitude and depth are measured in units of an ocean wavelength. The magnetic field vector above the surface is circularly polarized in a right-handed sense with respect to the unit vector $\hat{n}$, and its magnitude decreases exponentially with altitude. Immediately below the surface, the magnetic field vector is elliptically polarized with the major

* Expressions for a deep ocean approximation agree with results presented by Weaver [1965].

axis of the ellipse aligned vertically and with a right-handed sense of rotation. At a depth of $\lambda/4\pi$, the field is linearly polarized in a vertical direction. At greater depths, the field is again elliptically polarized with the major axis of the ellipse aligned vertically, but the sense of rotation is left-handed. At great depths, polarization becomes circular with a left-handed sense of rotation.

Maximum magnetic field strengths are at the surface, and values scale according to the square root of ocean wavelength. In equatorial regions, magnetic field strengths are less than field strengths in polar regions by a factor of $(1/2) \cos \theta$. At intermediate latitudes, field strengths are less than field strengths in polar regions by a factor of

$$R = \frac{1}{2}\left[(1 + 3 \sin^2\Phi)(\sin^2\phi + \cos^2\phi \cos^2\theta)\right]^{1/2} . \qquad (31)$$

In shallow seas or for wavelengths greater than an ocean depth, however, variations in polarization and magnetic field strength with depth change with latitude. Profiles of polarization and magnitude of the magnetic field induced in a shallow sea ($kD = 0.1$) by a one meter high surface wave propagating in polar and equatorial regions are shown as functions of z/D in Figure 4. We observe that profiles are symmetric in polar regions, because the horizontal component of sea water velocity is effectively independent of depth in a shallow sea. In equatorial regions, however, profiles are asymmetric, because the vertical component of sea water velocity vanishes at the ocean floor.

V. ELECTROMAGNETIC FIELDS GENERATED BY INTERNAL WAVES

Description of electromagnetic fields generated by internal waves is more complicated than for surface waves because internal waves are anistropic and propagate at oblique angles to the surface. As a result, numerous internal waves or modes contribute to the velocity field corresponding tc a particular horizontal wave number. Electromagnetic fields generated by internal waves, then, are sums of fields generated by each internal wave mode.

Because internal waves produce negligibly small surface displacements, we neglect the vertical component of velocity at the surface and find from Equations (26a) and (26b) that vector profiles of magnetic fields generated by an nth mode internal wave are given by

$$\vec{\beta}_n(z) = \delta\sqrt{2}\left\{(\vec{B}\cdot\hat{z})\nu_{zn}(z)-\sqrt{2}(\vec{B}\cdot\hat{\nu})\int_0^z \nu_{zn}(\zeta)e^{-k(z-\zeta)}kd\zeta\right\}\hat{\nu}$$

$$-\delta\sqrt{2}\left\{(\vec{B}\cdot\hat{z})\nu_{zn}(z)-\sqrt{2}(\vec{B}\cdot\hat{\nu}^*)\int_z^D \nu_{zn}(\zeta)e^{-k(\zeta-z)}kd\zeta\right\}\hat{\nu}^* \quad , \qquad (32a)$$

within the sea ($0 \le z \le D$), and by

$$\vec{\beta}_{vn}(h) = \frac{\mu_0\sigma}{k} e^{-kh}(\vec{B}\cdot\hat{\nu}^*)\nu^*\int_0^D \nu_{zn}(\zeta)e^{-k\zeta}kd\zeta \quad , \qquad (32b)$$

at a height h above the surface, where $\nu_{zn}(z)$ is the vertical component of the vector profile of velocity for an nth mode internal wave. Corresponding vector profiles of electric fields generated by an nth mode internal wave are given by

$$\vec{\epsilon}_n = \left(\frac{\omega_n}{k}\right)(\hat{z}\cdot\vec{\beta}_n)\hat{n} - (\vec{B}\cdot\hat{n})(\vec{\nu}_n \times \hat{n}) \quad , \tag{32c}$$

within the sea, and by

$$\vec{\epsilon}_{vn} = \left(\frac{\omega_n}{k}\right)(\hat{z}\cdot\vec{\beta}_{vn})\,\hat{n} \tag{32d}$$

above the surface, where $\omega_n(k)$ is the eigenfrequency of an nth mode internal wave corresponding to a horizontal wave number k. We note that the electrostatic part of the electric field vanishes above the surface, since the vertical component of velocity at the surface is negligible. We also observe that the magnetic field above the surface is proportional to a Laplace transform of the vertical component of the vector profile of velocity.

As we noted previously, velocity profiles corresponding to actual ocean stratification ordinarily must be obtained numerically or by suitable approximate methods. Here, we consider two model profiles that represent two extremes of ocean stratification: (1) an exponentially stratified ocean, for which the Brunt-Väisälä frequency profile is constant and (2) an ocean stratified in two layers, for which the Brunt-Väisälä frequency profile is represented by a delta function.

A. EXPONENTIAL STRATIFICATION

For a constant Brunt-Väisälä frequency, each internal wave mode is a sum of two internal waves having equal amplitudes and propagating obliquely to the surface at angles θ_n and $-\theta_n$, where $\tan\theta_n = n\pi/kD$,

with a dispersion given by the relation $\omega_n/N = \cos\theta_n$, which is equivalent to Equation (24c). The vector profile of velocity for an nth mode internal wave is then expressed in terms of circularly polarized components as

$$i\vec{\nu}_n = \frac{(-1)^n \nu_{ko,n}}{\sqrt{2}\sin\theta_n}\left\{\hat{\nu}\sin\left[n\pi(1-z/D)+\theta_n\right]\right.$$

$$\left. + \hat{\nu}^*\sin\left[n\pi(1-z/D)-\theta_n\right]\right\} , \qquad (33)$$

where $\nu_{ko,n}$ is the horizontal component of velocity at the surface owing to the nth mode.

Consequently, we find from Equations (32a) and (32b) that the vector profile of the magnetic field induced by an nth mode internal wave is given by

$$\vec{\beta}_n(z) = iA_n\left\{(\vec{B}\cdot\hat{\nu})\left[\sin\left(n\pi(1-z/D)+2\theta_n\right) - (-1)^n e^{-kz}\sin 2\theta_n\right]\right.$$

$$\left. - (\vec{B}\cdot\hat{\nu}^*)\sin n\pi(1-z/D)\right\}\hat{\nu} + iA_n\left\{(\vec{B}\cdot\hat{\nu})\sin n\pi(1-z/D)\right.$$

$$\left. - (\vec{B}\cdot\hat{\nu}^*)\left[\sin\left(n\pi(1-z/D)-2\theta_n\right) + e^{-k(D-z)}\sin 2\theta_n\right]\right\}\hat{\nu}^* \qquad (34a)$$

within the sea ($0 \le z \le D$), where

$$A_n = (-1)^n\left(\frac{\mu_o\sigma}{2k}\right)\nu_{ko,n}\left(\frac{kD}{n\pi}\right) , \qquad (34b)$$

and by

$$\vec{\beta}_{vn}(h) = i\nu_{ko,n} \frac{\mu_o\sigma}{k} \left[\frac{1 - (-1)^n e^{-kD}}{1 + (n\pi/kD)^2} \right] e^{-kh} (\vec{B}\cdot\hat{\nu}^*) \hat{\nu}^* \qquad (34c)$$

at a height h above the surface. Corresponding vector profiles of electric fields are given by Equations (32c) and (32d).

For purposes of illustration, we again choose an ocean depth of 1 km and consider induced magnetic fields alone. We present results in terms of magnetic field strengths generated per unit speed at the surface on a mode by mode basis, so that results are independent of the magnitude of the Brunt-Väisälä frequency. We note, however, that surface speed and the maximum vertical displacement of an nth mode internal wave, $\xi_{m,n}$, are related by the expression

$$\nu_{ko,n} = \frac{(-1)^{n+1} N \ \xi_{m,n}}{\sqrt{1 + (kD/n\pi)^2}} \quad , \qquad (35)$$

which results from the kinematic relation between vertical displacement and the vertical component of velocity. Field strengths generated per unit maximum displacement, then, are proportional to the Brunt-Väisälä frequency. Furthermore, high order modes produce greater surface speeds per unit maximum displacement than low order modes

Magnitudes of magnetic fields and corresponding horizontal gradients in the direction of wave propagation induced at the surface by internal waves having surface speeds of 1 cm/sec in a 1 km deep ocean in polar regions are shown as functions of kD by solid and dashed curves, respectively, in Figure 5 for modes 1 and 2. Magnitudes

of horizontal gradients correspond to the right-hand coordinate scale, marked in units of (gamma/m)/(cm/sec), and field strengths, to the left-hand coordinate scale, marked in units of gamma/(cm/sec). Field strengths generated per unit surface speed shown in Figure 5 scale according to ocean depth in kilometers, but magnitudes of horizontal gradients depend only on the ratio of ocean depth to wavelength, λ. At equatorial and intermediate latitudes, field strengths at the surface are less than field strengths in polar regions by the factor R as given by Equation (31).

We observe from Figure 5 and Equation (34c) that for small values of $n\pi/kD$, magnetic field strengths generated per unit surface speed are inversely proportional to wave number and so increase with decreasing wave number, because deeper eddy currents contribute to field strengths at the surface at smaller wave numbers. For large values of $n\pi/kD$, however, magnetic field strengths at the surface are proportional to wave number, for odd numbered modes, and to the square of wave number, for even numbered modes. At large values of $n\pi/kD$, sea water velocities of an nth mode internal wave are nearly linearly polarized and horizontally aligned with n phase reversals over an ocean depth. In polar regions then, where magnetic fields are generated by the horizontal component of sea water velocity, magnetic field strengths generated at the surface decrease with wave number because contributions of deep eddy currents increasingly oppose contributions of shallow eddy currents. In equatorial regions, however, where magnetic fields are generated by the vertical component of sea water velocity, magnetic field strengths generated at the surface decrease with wave number both because contributions of deep eddy currents increasingly oppose contributions of shallow eddy currents for all modes except the fundamental mode and because the

vertical component of velocity decreases with wave number for all modes as sea water velocity approaches linear polarization.

Profiles of magnitude and polarization of magnetic fields induced by a fundamental mode internal wave ($n = 1$) having a surface speed of 1 cm/sec and propagating in a 1 km deep ocean in polar regions and in equatorial regions, along magnetic meridians, are delineated in Figure 6, for $kD = 1$. We observe that profiles of field magnitude and polarization are symmetrical, as are velocity profiles. The major axis of polarization ellipses for odd numbered modes is horizontal in polar regions and vertical in equatorial regions. For even numbered modes, however, the major axis is vertical in polar regions and horizontal in equatorial regions. In each case, fields are linearly polarized at a depth of D/2. At intermediate latitudes, profiles of field magnitude and polarization are superpositions of profiles corresponding to polar and equatorial regions, with relative proportions depending on latitude. Nonetheless, polarization is always circular both above the ocean surface and below the ocean floor, and field magnitudes decrease exponentially as e^{-kh} with height above the surface and as $e^{-k(z-D)}$ with distance below the floor.

B. TWO-LAYER STRATIFICATION

In an ocean stratified in two layers, which is a model commonly used to represent a sharp thermocline, only a single internal wave mode propagates. Profiles of the vertical component of velocity corresponding to internal waves propagating on a sharp thermocline are expressed by Equation (25b). From Equations (32a) and (32b), we

find that the vector profile of the magnetic field induced by an internal wave propagating on a sharp thermocline at a depth d in an ocean of depth D is expressed by

$$\vec{\beta}_1 = iA_1 \left[(\vec{B}\cdot\hat{\nu}^*) \sinh kz + (\vec{B}\cdot\hat{\nu}) kze^{-kz} \right] \hat{\nu}$$

$$+ iA_1 \Bigg\{ (\vec{B}\cdot\hat{\nu}^*) \left[\frac{k(d-z)e^{-k(d-z)} \sinh kD - k(D-z)e^{-k(D-z)} \sinh kd}{\sinh k(D-d)} \right]$$

$$- (\vec{B}\cdot\hat{\nu}) \sinh kz \Bigg\} \hat{\nu}^* \quad , \qquad (36a)$$

above the thermocline ($0 \le z \le d$), where

$$A_1 = \nu_{ko} \frac{\mu_o \sigma}{2k} \quad , \qquad (36b)$$

by

$$\vec{\beta}_2 = iA_2 \Bigg\{ (\vec{B}\cdot\hat{\nu}^*) \sinh k(D-z)$$

$$+ (\vec{B}\cdot\hat{\nu}) \left[\frac{kze^{-kz} \sinh k(D-d) - k(z-d)e^{-k(z-d)} \sinh kD}{\sinh kd} \right] \Bigg\} \hat{\nu}$$

$$- iA_2 \left[(\vec{B}\cdot\hat{\nu}^*) k(D-z)e^{-k(D-z)} + (\vec{B}\cdot\hat{\nu}) \sinh k(D-z) \right] \hat{\nu}^* \quad , \qquad (36c)$$

below the thermocline ($d \le z \le D$), where

$$A_2 = A_1 \frac{\sinh kd}{\sinh k(D-d)} \quad , \tag{36d}$$

and by

$$\vec{\beta}_v = i\nu_{ko}\left(\frac{\mu_o \sigma d}{2}\right)\left[1 - \frac{(D/d - 1)e^{-kD} \sinh kd}{\sinh k(D-d)}\right] e^{-kh}(\vec{B}\cdot\hat{\nu}^*)\hat{\nu}^* \tag{36e}$$

at a height h above the surface.* Corresponding vector profiles of electric fields are expressed by

$$\vec{\epsilon}_1 = \frac{\omega}{k}(\hat{z}\cdot\vec{\beta}_1)\hat{n} - \nu_{ko}\frac{(\vec{B}\cdot\hat{n})}{\sqrt{2}}(e^{-kz}\hat{\nu} + e^{kz}\hat{\nu}^*) \quad , \tag{37a}$$

above the thermocline, by

$$\vec{\epsilon}_2 = \frac{\omega}{k}(\hat{z}\cdot\vec{\beta}_2)\hat{n} + \frac{\nu_{ko} \sinh kd}{\sinh k(D-d)} \frac{(\vec{B}\cdot\hat{n})}{\sqrt{2}}[e^{k(D-z)}\hat{\nu}$$

$$+ e^{-k(D-z)}\hat{\nu}^*] \quad , \tag{37b}$$

below the thermocline, and by

$$\vec{\epsilon}_v = \frac{\omega}{k}(\hat{z}\cdot\vec{\beta}_v)\hat{n} \quad , \tag{37c}$$

at a height h above the surface, with a phase speed determined by the dispersion relation expressed by Equation (25c).

For purposes of illustration, we choose a thermocline depth of 100 m and an ocean depth of 1 km and, as before, consider induced

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* Expressions for the vector profile of the magnetic field corresponding to a deep ocean approximation (kD>>1) agree with results presented by Beal and Weaver, [1970].

magnetic fields alone. We present results in terms of magnetic field strengths generated per unit vertical displacement, ξ_d, of the thermocline. Surface speed and thermocline displacement are related by the expression

$$\nu_{ko} = \xi_d \sqrt{\frac{g}{d}\left(\frac{\Delta\rho}{\rho}\right)} \left[\frac{kd \sinh k(D-d)}{\sinh kd \sinh kD}\right]^{1/2} , \tag{38}$$

owing to the kinematic relation between vertical displacement and the vertical component of velocity. We choose a representative value of 0.001 for the fractional increase in density across a thermocline.

Magnitudes of magnetic fields and horizontal gradients in the direction of wave propagation generated at the surface by internal waves producing a 1 m displacement of a sharp thermocline at a depth of 100 m in a 1 km deep ocean in polar regions are shown as functions of kD in Figure 7 by solid and dashed curves, respectively. For a fixed ratio of thermocline depth to ocean depth, magnetic field strengths per unit thermocline displacement shown in Figure 7 scale according to the square root of thermocline depth, but magnitudes of horizontal gradients scale according to the inverse square root of thermocline depth. At equatorial and intermediate latitudes, magnitudes of magnetic fields and field gradients at the surface are less than values in polar regions by the factor R expressed by Equation (31).

As is evident from Figure 7, magnetic field strengths generated per unit thermocline displacement increase with decreasing wave number at large wave numbers ($kD \gg 1$), because deeper eddy currents contribute to field strengths at the surface at smaller wave numbers. For small wave numbers ($kD \ll 1$), however, field strengths generated at the surface decrease in proportion to wave number, as is evident from Equation (36e). At small wave numbers, sea water

velocities correspondint to an internal wave propagating on a sharp thermocline are nearly linearly polarized and horizontally aligned with opposite phases above and below the thermocline. In polar regions, then, magnetic field strengths generated at the surface decrease with wave number because contributions of eddy currents below the thermocline increasingly oppose contributions of eddy currents above the thermocline as wave number decreases. In equatorial regions, however, magnetic field strengths generated at the surface decrease with wave number because the vertical component of velocity decreases with wave number as sea water velocity approaches linear polarization.

Profiles of magnetic field magnitude and polarization in polar and equatorial regions are shown in Figure 8, for wavelengths much greater than a thermocline depth corresponding to $kD = 0.5$. In polar regions, the major axis of polarization ellipses is horizontal near the thermocline and becomes vertical away from the thermocline. Three polarization reversals occur in polar regions: one just below the surface, another just below the thermocline, and the last just above the bottom. For wavelengths much less than a thermocline depth, profiles are symmetrical with a polarization reversal at the thermocline and one directly above and below the thermocline. In equatorial regions, the major axis of polarization ellipses is vertical, and a single polarization reversal occurs directly below the thermocline. Profiles are again symmetrical for wavelengths much less than a thermocline depth with the polarization reversal at the thermocline. At intermediate latitudes, profiles of field magnitude and polarization again are superpositions of profiles corresponding to polar and equatorial regions, with relative proportions depending on latitude. As always, polarization is circular both above and below an ocean, and field magnitudes decrease exponentially as e^{-kh} with height above the surface and as $e^{-k(z-D)}$ with distance below the floor.

CONCLUSION

As we have demonstrated, the gradient in a direction $\hat{u}$ of the magnetic field, $\vec{b}$, generated at a height h above an ocean surface by a horizontally progressing ocean is expressed by

$$(\hat{u}\cdot\vec{\nabla})\,\vec{b} = -i\mu_0\sigma e^{-kh}(\hat{u}\cdot\hat{\nu}^*)\hat{\nu}^*\int_0^D \hat{n}\cdot(\vec{V}\times\vec{B})\,e^{-k\zeta}\,k\,d\zeta \quad , \qquad (39)$$

where $\vec{V}$ is the velocity field of the wave. As is evident from Equation (39), gradients of magnetic fields generated above the surface by progressive waves are circularly polarized in a vertical plane containing the direction of wave propagation, and their magnitudes decrease exponentially with height above the surface. Magnitudes of gradients at the surface are proportional to a weighted average of sea water speed over an ocean depth.

Because magnitudes of magnetic field gradients induced at [illegible]e surface are proportional to a weighted average of sea water speed [illegible]er an ocean depth, field gradients respond to the mode structure of [illegible]nal waves, as is also evident from Figure 5. Moreover, because [illegible]tudes of field gradients decrease exponentially with height above the surface as e^{-kh}, vertically spaced measurements above the surface furnish means of wave number discrimination, which together with frequency discrimination affords means of resolving spectra on a mode by mode basis. Finally, noise spectra of magnetic gradients, in effect, are cross-spectra and so provide directional information on wave spectra from an effective point measurement. The imaginary part of the spectrum, for example, vanishes for isotropic wave spectra. Either rotation of gradiometer axes or multiple sets of axes provide information on anisotropy of a wave field.

As is evident from Figures 1 and 5, gradients of magnetic fields generated at an ocean surface by both surface and internal waves are of the order of 10^{-3} gamma/m for surface speeds of 1 cm/sec, and so provide sensible signals for recently developed superconducting, magnetic gradiometers at surface speeds typically produced by ocean waves. Magnetic field gradients produced at an ocean surface by fluctuating ionospheric currents are of the order of 10^{-4} gamma/m or less, so that magnitudes of gradients generated by ocean waves exceed noise levels produced by ionospheric currents by a factor of ten or more for typical surface speeds. Measurements of fluctuating magnetic field gradients at a fixed point above an ocean surface, then, provide an estimate of ocean wave spectra.

Nonetheless, a hallmark of oceanographic observations is the absence of a stationary platform. Use of a so-called transverse axis gradiometer (in which the two pickup loops lie in one plane) to record fluctuating gradients of field components transverse to Earth's magnetic field, however, suppresses apparent gradient fluctuations owing to changes in gradiometer orientation. By aligning the gradiometer axis so that a line of force of Earth's field passes through the center of each pickup loop, changes in gradiometer orientation then produce apparent gradient fluctuations that are of the order of the product of Earth's magnetic field gradient ($\sim 10^{-2}$ gamma/m) and the sine of the angular deviation in alignment, or about 10^{-5} gamma/m for a deviation of one milliradian.

Thus, we submit that using appropriately designed, superconducting, magnetic gradiometers to make vertically spaced measurements of magnetic field gradients generated above the surface by ocean waves provides a unique and promising means of obtaining a measure of ocean wave spectra -- for both surface and internal waves.

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FIGURE CAPTIONS

Figure 1. Magnitudes of magnetic fields and horizontal gradients in the direction of wave propagation generated at the surface of a 1 km deep ocean by surface waves having horizontal surface speeds of 1 cm/sec in polar regions and vertical surface speeds of 1 cm/sec in equatorial regions and propagating along magnetic meridians in equatorial regions shown as functions of kD along the lower abscissa and as functions of D/λ along the upper abscissa. Solid curves delineate magnetic field strengths per unit surface speed and correspond to the left-hand coordinate scale marked in units of gamma/(cm/sec). Dashed curves delineate magnitudes of horizontal gradients along the direction of wave propagation per unit surface speed and correspond to the right-hand coordinate scale marked in units of (gamma/m)/(cm/sec).

Figure 2. Magnitudes of magnetic fields and horizontal gradients in the direction of wave propagation generated at the surface of a 1 km deep ocean by a one meter high surface wave propagating in polar regions ($\Phi = 90°$) and along magnetic meridians at intermediate and equatorial latitudes ($\Phi = 45°$ and $0°$) shown as functions of kD along the lower abscissa and as functions of D/λ along the upper abscissa. Solid curves delineate magnetic field strengths per unit wave height and correspond to the left-hand coordinate scale marked in units of gamma/m. Dashed curves delineate magnitudes of horizontal gradients along the direction of wave propagation per unit wave height and correspond to the right-hand coordinate scale marked in units of (gamma/m)/m.

Figure 3. Profiles of polarization and magnitude of the magnetic field generated in a deep ocean ($kD \gg 1$) by a one meter high surface wave having a wavelength of 10 m and propagating in polar regions. Altitude and depth are measured in units of an ocean wavelength.

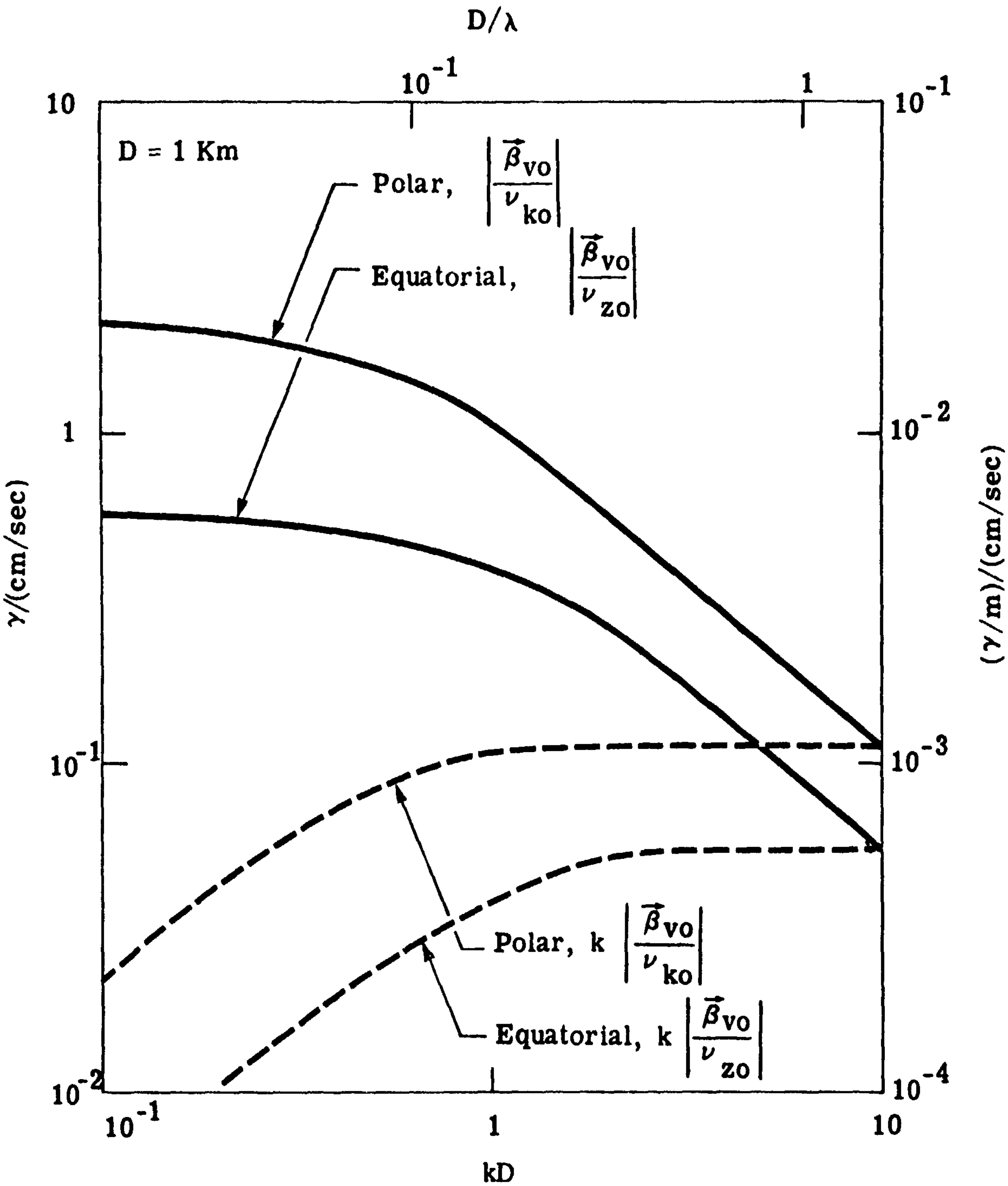
Figure 4. Profiles of polarization and magnitude of the magnetic field generated in a 1 km deep ocean by a one meter high surface wave propagating in polar regions and along magnetic meridians in equatorial regions and having a wavelength corresponding to $kD = 0.1$. Altitude and depth are measured in units of an ocean depth, D.

Figure 5. Magnitudes of magnetic fields and corresponding horizontal gradients in the direction of wave propagation generated at the surface of a 1 km deep ocean in polar regions by internal wave modes 1 and 2 each having surface speeds of 1 cm/sec, shown as functions of kD along the lower abscissa and as functions of D/λ along the upper abscissa. Solid curves delineate magnetic field strengths per unit surface speed and correspond to the left-hand coordinate scale marked in units of gamma/(cm/sec). Dashed curves delineate magnitudes of horizontal gradients in the direction of wave propagation and correspond to the right-hand coordinate scale marked in units of (gamma/m)/(cm/sec).

Figure 6. Profiles of polarization and magnitude of the magnetic field generated in a 1 km deep ocean by a fundamental mode internal wave ($n=1$) propagating in polar regions and along magnetic meridians in equatorial regions and having a surface speed of 1 cm/sec and a wavelength corresponding to $kD = 1$. Altitude and depth are measured in units of an ocean depth.

Figure 7. Magnitudes of magnetic fields and horizontal gradients in the direction of wave propagation generated at the surface of a 1 km deep ocean by internal waves producing a 1 m displacement of a sharp thermocline at a depth of 100 m in polar regions shown as functions of kD along the lower abscissa and as functions of D/λ along the upper abscissa. Solid curves delineate magnetic field strengths per unit displacement and correspond to the left-hand coordinate scale marked in units of gamma/m. Dashed curves delineate magnitudes of horizontal gradients in the direction of wave propagation and correspond to the right-hand coordinate scale marked in units of (gamma/m)/m.

Figure 8. Profiles of polarization and magnitude of the magnetic field generated in a 1 km deep ocean by an internal wave propagating in polar regions and along magnetic meridians in equatorial regions and producing a 1 m displacement of a sharp thermocline at a depth of 100 m at a wavelength corresponding to kD = 0.5. Altitude and depth are measured in units of a thermocline depth.



D/λ
10^{-1}
1
10
1
10^{-1}
10^{-2}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
D = 1 Km
Polar, $\left|\frac{\vec{\beta}_{vo}}{\nu_{ko}}\right|$
Equatorial, $\left|\frac{\vec{\beta}_{vo}}{\nu_{zo}}\right|$
Polar, k $\left|\frac{\vec{\beta}_{vo}}{\nu_{ko}}\right|$
Equatorial, k $\left|\frac{\vec{\beta}_{vo}}{\nu_{zo}}\right|$
γ/(cm/sec)
(γ/m)/(cm/sec)
10^{-1}
1
10
kD

Figure 1

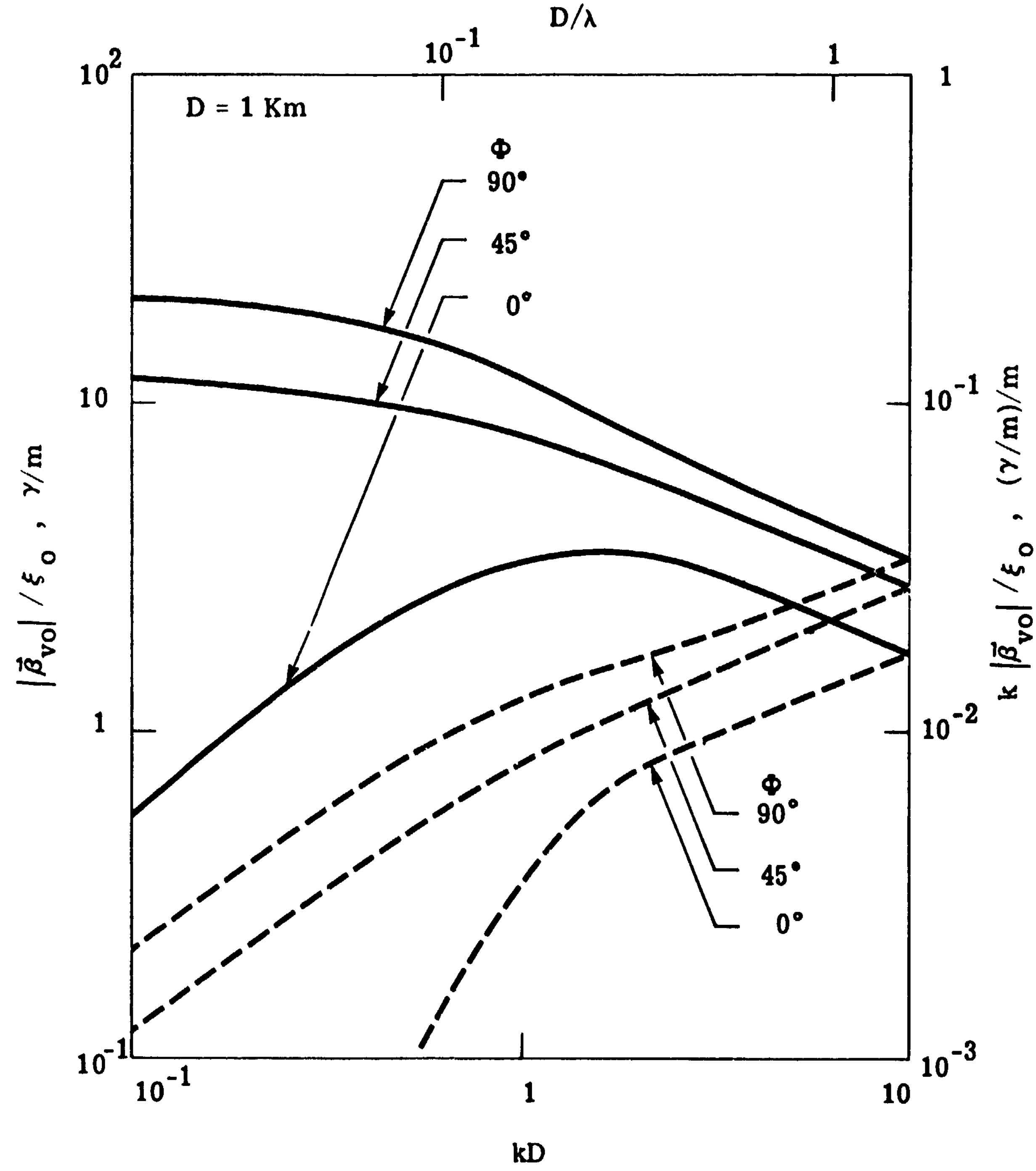
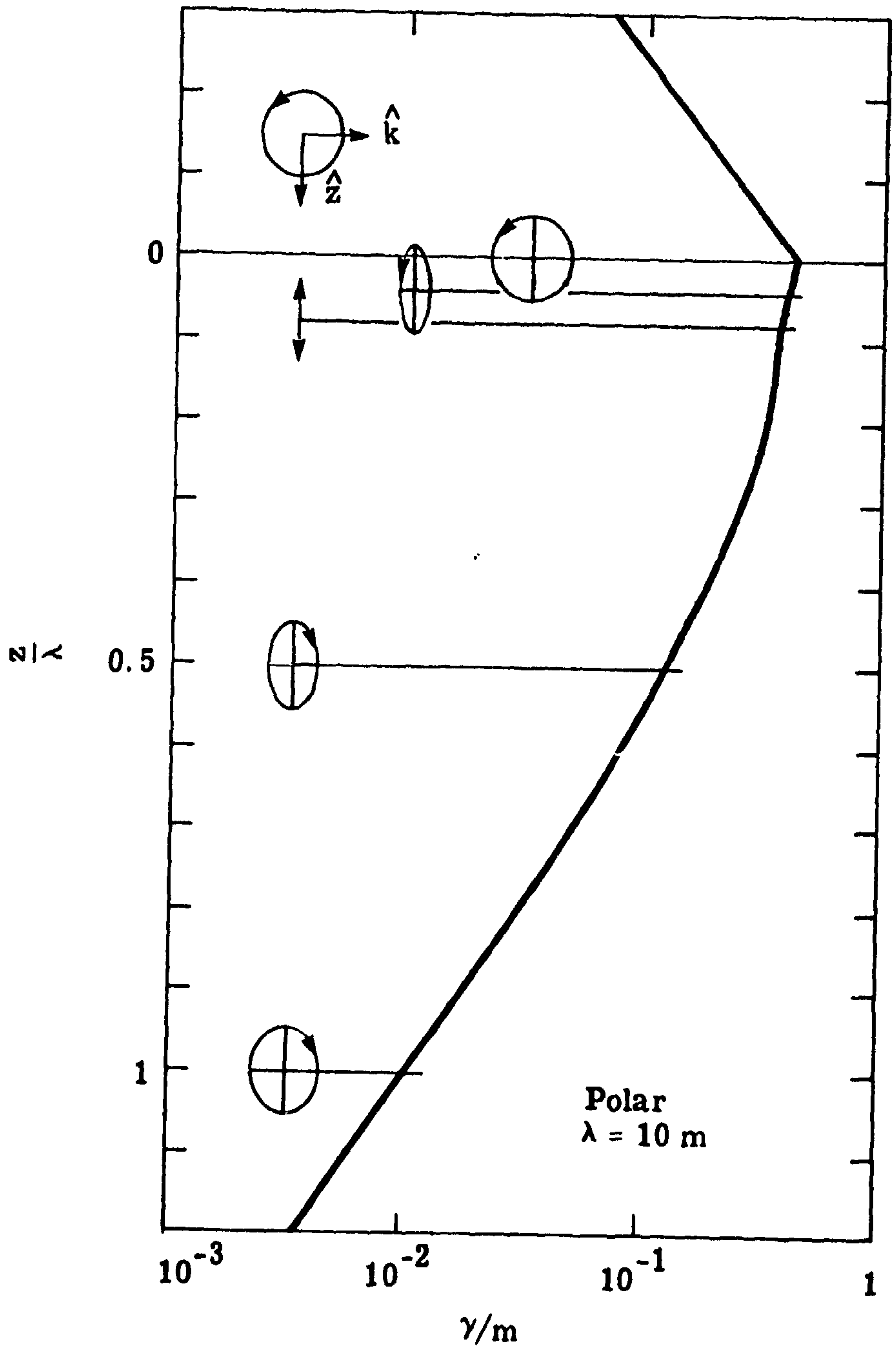

D/λ
10^{-1}
1
10^2
1
D = 1 Km
Φ
90°
45°
0°
10
10^{-1}
$|\vec{\beta}_{vo}| / \xi_o$, γ/m
$k\,|\vec{\beta}_{vo}| / \xi_o$, (γ/m)/m
1
10^{-2}
Φ
90°
45°
0°
10^{-1}
10^{-3}
10^{-1}
1
10
kD

Figure 2



$\hat{k}$
$\hat{z}$
0
$\frac{z}{\lambda}$
0.5
1
Polar
λ = 10 m
10^{-3}
10^{-2}
10^{-1}
1
γ/m

Figure 3

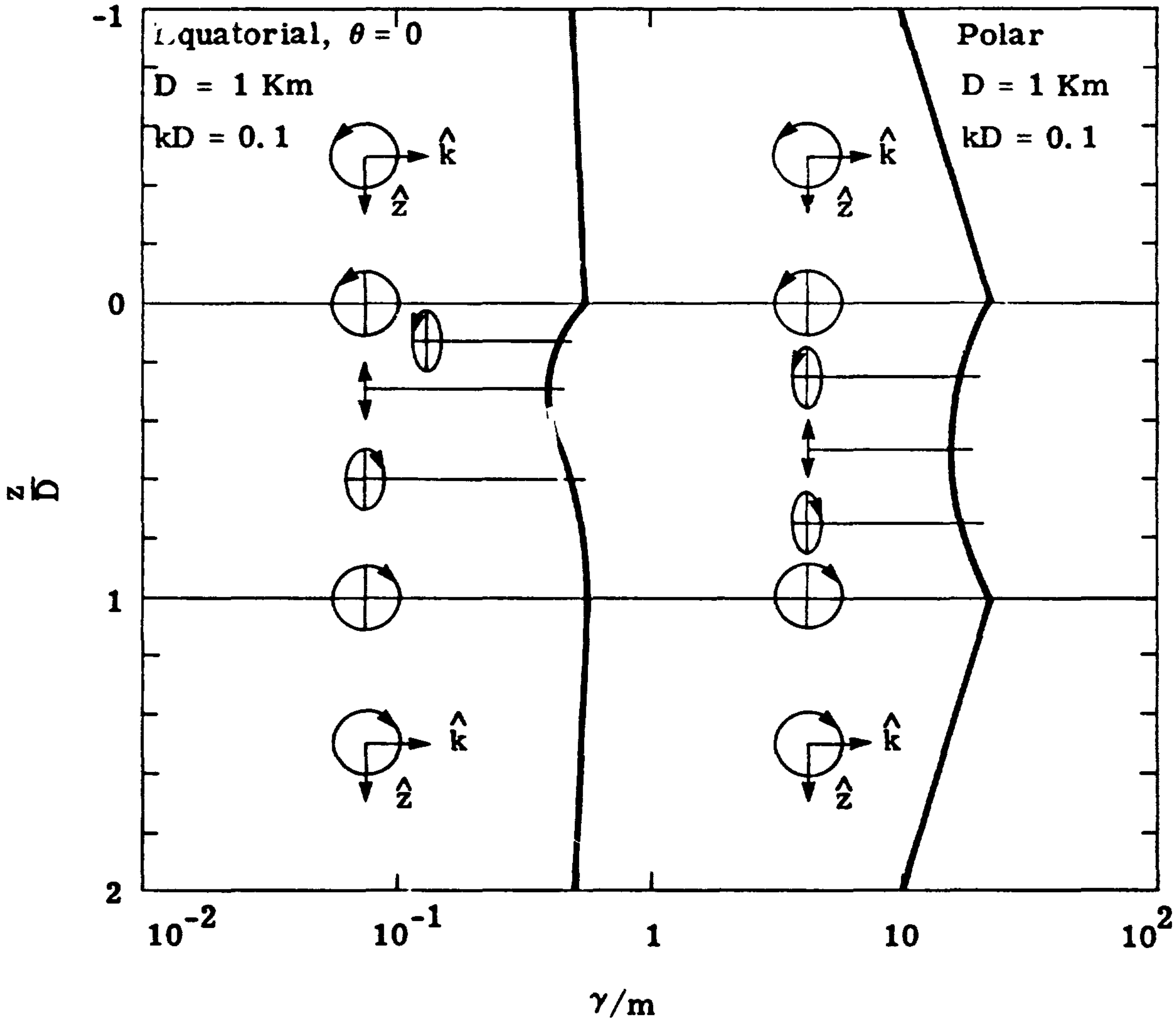
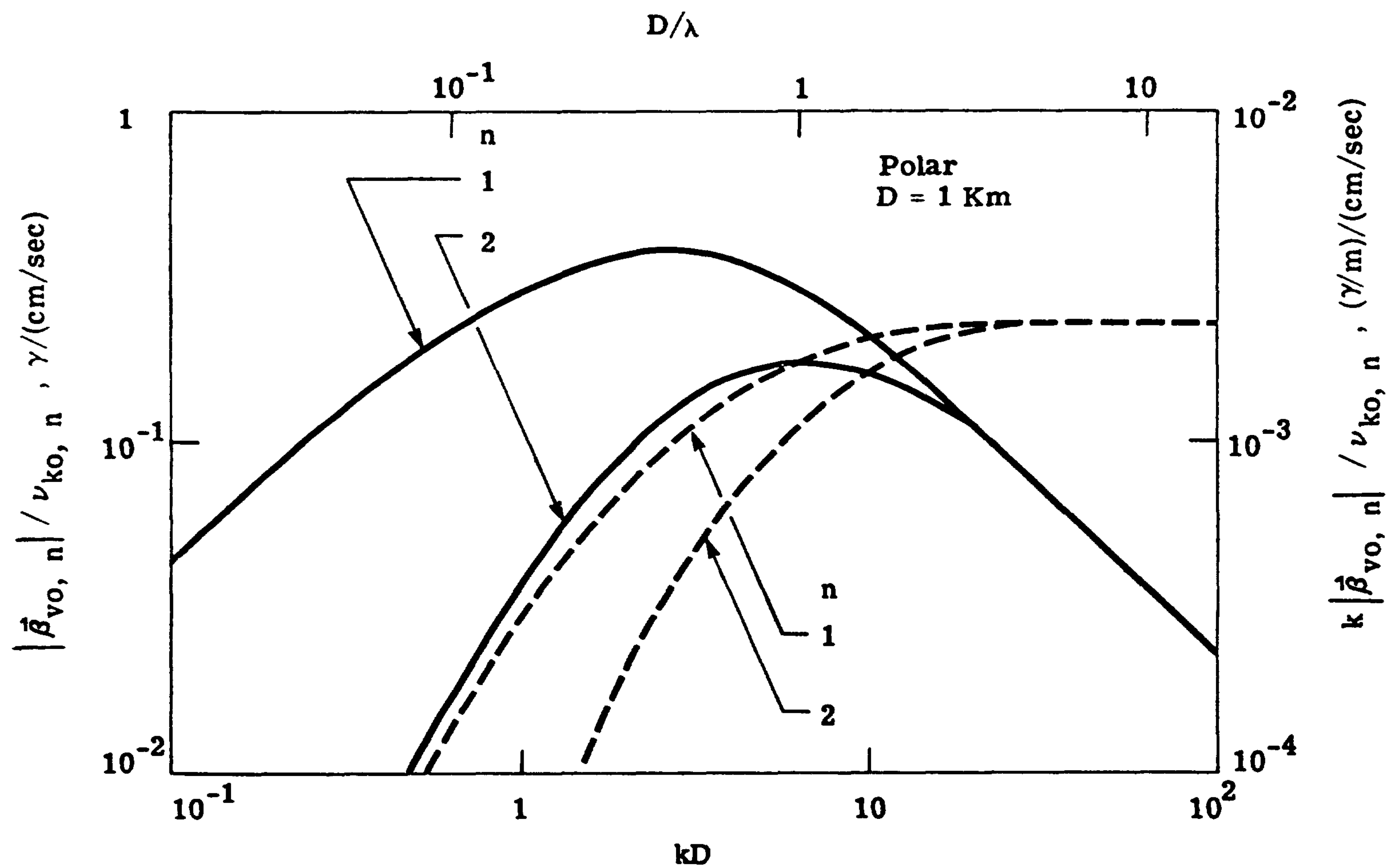
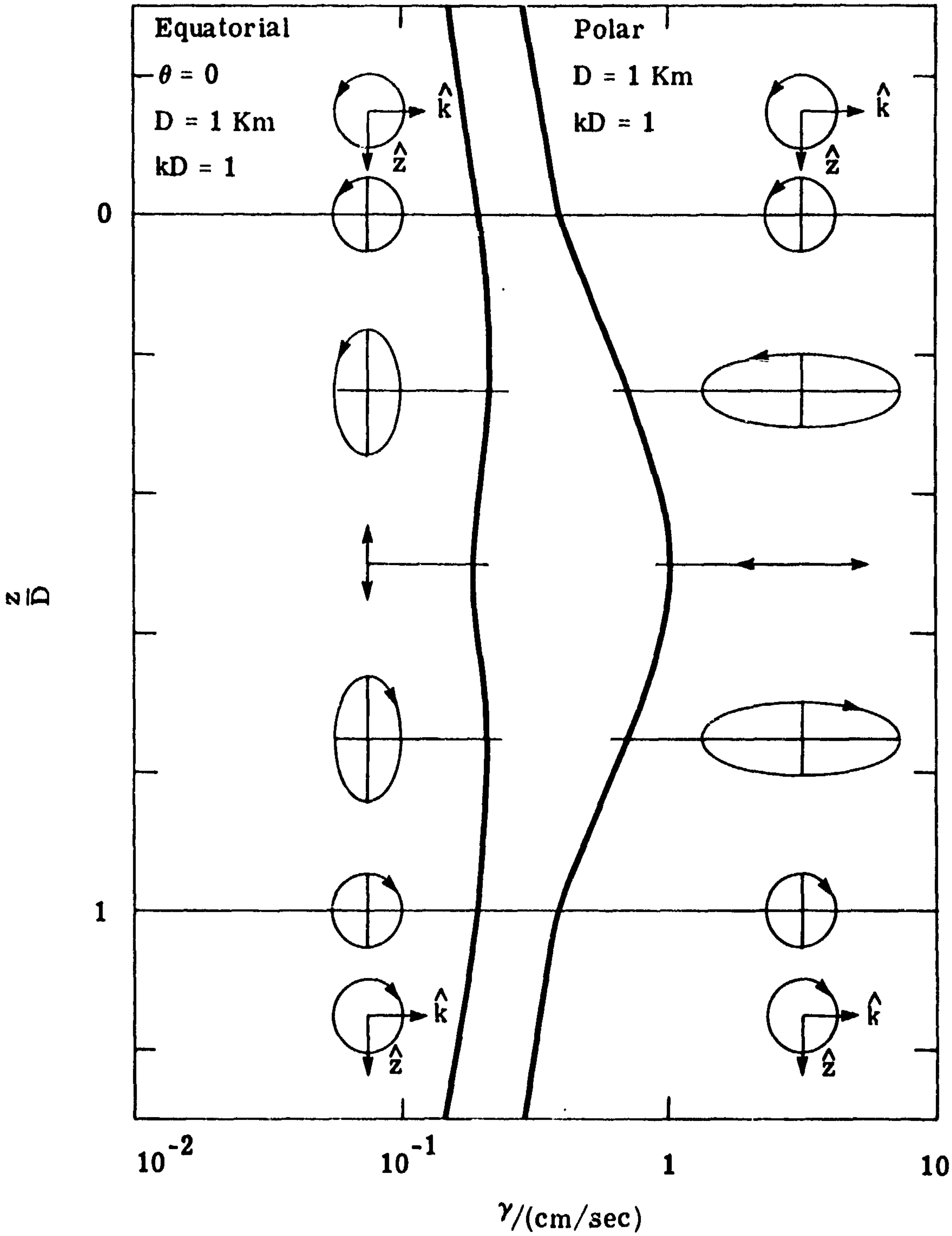

Equatorial, $\theta = 0$
D = 1 Km
kD = 0.1
Polar
D = 1 Km
kD = 0.1
$\hat{k}$
$\hat{z}$
-1
0
1
2
$\frac{z}{D}$
10^{-2}
10^{-1}
1
10
10^{2}
γ/m

Figure 4



D/λ
10^{-1}
1
10
Polar
D = 1 Km
n
1
2
n
1
2
$|\hat{\beta}_{vo,\,n}| / \nu_{ko,\,n}$, γ/(cm/sec)
$k\,|\hat{\beta}_{vo,\,n}| / \nu_{ko,\,n}$, (γ/m)/(cm/sec)
1
10^{-1}
10^{-2}
10^{-2}
10^{-3}
10^{-4}
10^{-1}
1
10
10^{2}
kD

Figure 5



Equatorial
$\theta = 0$
D = 1 Km
kD = 1
Polar
D = 1 Km
kD = 1
$\hat{k}$
$\hat{z}$
0
1
$\frac{z}{D}$
10^{-2}
10^{-1}
1
10
γ/(cm/sec)

Figure 6

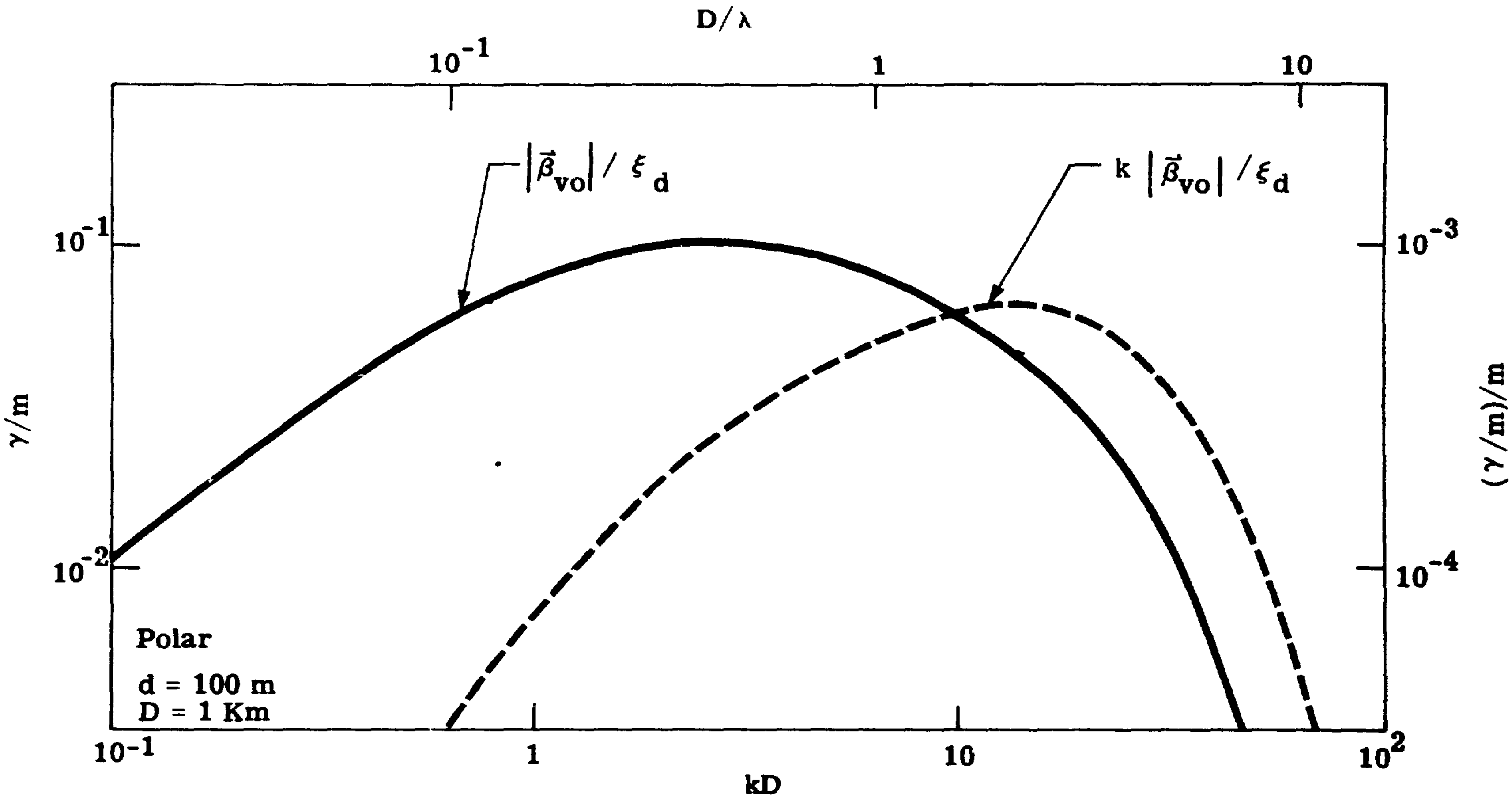
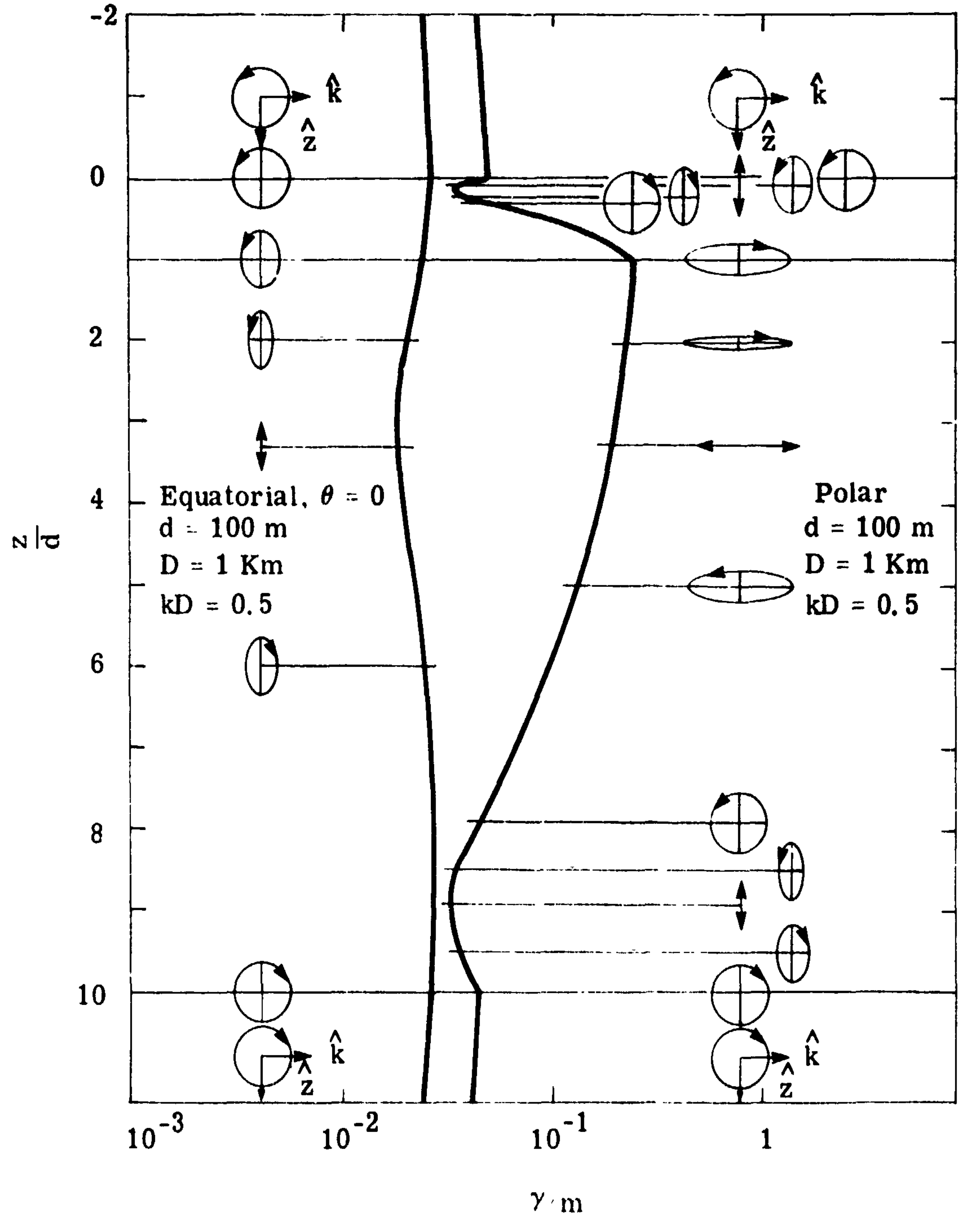

D/λ
10^-1
1
10
10^-1
10^-2
10^-3
10^-4
γ/m
(γ/m)/m
$|\vec{\beta}_{vo}| / \xi_d$
$k\ |\vec{\beta}_{vo}| / \xi_d$
Polar
d = 100 m
D = 1 Km
10^-1
1
10
10^2
kD

Figure 7



-2
0
2
4
6
8
10
$\hat{k}$
$\hat{z}$
Equatorial, $\theta = 0$
d = 100 m
D = 1 Km
kD = 0.5
Polar
d = 100 m
D = 1 Km
kD = 0.5
$\frac{z}{d}$
10^{-3}
10^{-2}
10^{-1}
1
γ, m

Figure 8